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SOUND PROPAGATION IN A TURBULENT ATMOSPHERE*

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In the present report, we deal with the problem of explaining (from the viewpoint of the statistical theory of turbulence) certain results of experiments on the observation of the fluctuations in the phase of a sound wave during the propagation of sound in a turbulent atmosphere (Ref. 1).

We represent the atmospheric turbulence in such a way that all the space in the area of interest to us between the loudspeaker and the microphone is filled with heterogeneities of a field of velocities, superimposed one upon the other.

Let us consider a case when we have a loudspeaker and two microphones located at a certain distance (base) from one another. Let us direct the x-axis from the loudspeaker to the center of the base B(B $\perp$ x). If c = the speed of sound, ω = the angular frequency, v = the pulsation velocity of the wind, and L = the distance between the loudspeaker and the center of the base, the phase variation for the base center, allowing for the effect of wind heterogeneity on the path of propagation P – M, will be

$$\varphi = \omega\int_0^L \frac{dx}{c+v} \simeq \left(\frac{\omega}{c^2}\right)\int_0^L v\,dx + \text{const.}$$

Here we disregard the square of the ratio v/c, i.e. we consider only the wind velocity components in the direction of the sound propagation.

Considering the angle θ between the x-axis and the radial line PM_1 (PM_2) to be small, for the phase difference between the microphones, we will have:

$$\psi = \varphi_{M_1} - \varphi_{M_2} = \frac{\omega}{c^2}\int_0^L (v_1 - v_2)\,dx$$

and

$$\overline{\psi^2} = \left(\frac{\omega}{c^2}\right)\int_0^L\int_0^L \overline{\Delta v_1 \Delta v_2}\,dx_1\,dx_2.$$

* Translation of "O rasprostranenii zvuka v turbulentnoy atmosfere." Doklady Akademii Nauk SSSR, Vol. 47, No. 7, 486-9, 1945. Translated on 28 July 1964 by L.G. Robbins, ATSS-T.

For taking into account the correlation of pulsations in various points of the flow, we make use of the "2/3 law", obtained simultaneously by various methods by A.N. Kolmogorov (Ref. 2) and A.M. Obukhov (Ref. 3). According to this law, the square of the difference in velocities in pulsations at points 1, 2 /487 is proportional to the distance between these points to the power of 2/3:

$$\overline{(v_1 - v_2)^2} = C^2 r_{1,2}{}^{2/3}, \qquad (1)$$

where C = the characteristic of turbulence, having a dimensionality $cm^{2/3} sec^{-1}$, and $r_{1,2}$ = the distance between points 1, 2*. Based on the proposed isotropicity of turbulence, $\overline{v_1^2} = \overline{v_2^2}$ and $\overline{v^2} - \overline{v_1 v_2} = 1/2\, C^2 r_{1,2}^{2/3}$. Having used this relationship, we get for $\overline{\Delta v_1 \, \Delta v_2}$ the following expression:

$$\overline{\Delta v_1 \Delta v_2} = C^2 \left\{ [(x_1 - x_2)^2 + \theta^2 (x_1 + x_2)^2]^{1/3} - (x_1 - x_2)^{2/3} (1 + \theta^2)^{1/3} \right\},$$

where x_1 and x_2 are coordinates of points 1, 2 along the radial line.

The value for $\overline{\psi^2}$ is written in the form:

$$\overline{\psi^2} = \left(\frac{\omega}{c^2}\right)^2 C^2 \int_0^L \int_0^L \{ [(x_1 - x_2)^2 + \theta^2 (x_1 + x_2)^2]^{1/3} - - (x_1 - x_2)^{2/3} (1 + \theta^2)^{1/3} \} \, dx_1 dx_2.$$

As a result of the calculation of this integral, we get(for the mean square fluctuation of phase) the following formula:

$$\sigma\varphi = \sqrt{\overline{\psi^2}} = \mathrm{const} \left(\frac{\omega}{c^2}\right) C L^{1/3} B^{4/3}, \qquad (2)$$

where const has the value of the order of unity.

It should be noted that the adduced results are valid for the case of geometric acoustics. At low frequencies, when there comes into play the diffraction of sound on the heterogeneities, the dependence on frequency can be different and $\sigma\varphi$ will evidently tend more quickly toward zero.

There was established experimentally a dependence of the phase fluctuations on wind speed (Ref. 1) in the frequency range of 2,000-5,000 cps that was very close to direct proportionality. This fact can be explained on the basis of the derived formula (2), if we postulate that the turbulence characteristic C is directly proportional to the wind speed. Such a dependence is quite natural from the viewpoint of the statistical theory of turbulence and is in conformity with the results obtained from the measurements of turbulence in the near-ground layer (Refs. 4, 5).

* We take into consideration only the mean square of the longitudinal difference in velocities of pulsations (B_{dd} in the notations used by A.N. Kolmogorov [Ref. 2]), considering the correction owing to the mean square of transverse difference (B_{nn}) to be small.

The dependence of $\sigma\varphi$ upon ω and on the base B as $B^{5/6}$ also agrees with the experiment.

The expounded theory of the phase fluctuations can be applied for the calculation of the mean quadratic phase variability. In this connection, it is necessary to make the considerable assumption that the variations (for the brief time interval Δt) of the turbulence field in the system of coordinates, moving at the mean wind speed, are small in comparison with the local variations in the turbulence, evoked by the transport of the entire pattern with the mean velocity of the wind.

Let us consider a case when we have a loudspeaker and a microphone, located at the distance L from each other, and the wind direction is perpendicular to the line PM. In distinction from the case analyzed above with two microphones, here the wind direction is very important. Using relationship (1), after a number of calculations, we get the following formula for the mean quadratic variability of the phase

$$\sigma\Delta_t\varphi = \sqrt{\overline{\Delta_t\varphi^2}} = \sqrt{3LC}\,l^{5/6}\left(\frac{\omega}{c^2}\right), \tag{3}$$

where $l = v\,\Delta t$.

If we take from the experiment the values occurring in this formula, we can then find C. In the experiment, there occurred: $\sigma\Delta_t\varphi = 18°$; $\Delta t = 0.1$ sec; L = 140 m; f = 800 cps, $\overline{v}$ = 6.5 m/sec. From this C = 10.9 $cm^{2}/_{3}sec^{-1}$.

The C-value computed from the Goedicke experiments (Ref. 4) at a height of 1.15 m from the soil ($\overline{v}$ = 0.65 m/sec) amounts to C = 2.05. The value for C = 10.9 obtained from Eq. (3) is quite comparable in order of magnitude with the values for C also obtained by other methods, if in this connection we consider the difference in wind speed. A very close coincidence in the indicated case would be difficult to expect in view of the difference in height of the layer being studied and in the nature of the local conditions.

In our opinion, the results obtained can be of interest for an explanation of the source of errors during the functioning of sound receivers. Let us assume that the direction to the sound source forms the angle α with the direction of the base. Then in an unperturbed flow, the phase difference at the ends of the base will be:

$$\varphi = \frac{2\pi B}{\Lambda}\cos\alpha,$$

where Λ = the wave length. If now under the effect of turbulence, the observed phase receives the random variation $\delta\alpha$, then:

$$\delta\alpha = \frac{\Lambda}{2\pi B}\frac{1}{\sin\alpha}\delta\varphi.$$

At high α -values: $$\delta\alpha = \frac{\Lambda}{2\pi B}\delta\varphi.$$

A similar relationship will occur for the mean square errors:

$$\sigma\alpha = \frac{\Lambda}{2\pi B}\sigma\varphi,$$

or, if we use Eq. (2):

$$\sigma\alpha = \frac{\text{const}}{2\pi} C L^{1/2} B^{-1/6}.$$

From this equation it follows that $\sigma\alpha$ does not depend on the length of the sound wave Λ and depends to a very slight degree on the base. If we use the experimental data, we can find the constant in this formula. Let us take a case when B = 1 m, L = 20 m, $\overline{v}$ = 2.7 m/sec, f = 4,000 cps, Λ = 8.5 cm, and $\sigma\varphi = 24°$. At these data, const ~ 2. If we now consider that $\sigma\varphi$, and hence $\sigma\alpha \sim \overline{v}$ and that in the experiment under study $\overline{v}$ = 2.7 m/sec and L = = 20 m, we will then get the following formula for the mean square angular error of the sound detector, occurring owing to the turbulence:

$$\sigma\alpha^{0} \cong 0{,}3 B^{-1/6}\left(\frac{L}{20}\right)^{1/2}\frac{\overline{v}}{2{,}7}, \qquad (4)$$

where B, L, and $\overline{v}$ are given in meters. If e.g. we take B = 1 m and v= 2.7 m/sec, for L = 2 km, $\sigma\alpha \cong 3°$, i.e. in the given case, the error during the direction finding of the target will be about 104 m.

For the low frequencies of the order of 200 - 500 cps, usually comprising the working range of the sound detectors, this equation needs refinement, since at such frequencies, we can by no means consider it valid to use the conclusions of geometric acoustics alone. It is known that the sound detectors function poorly during bad weather conditions and especially during a strong wind. From the viewpoint discussed above, this is explained to a considerable degree by the effect of the atmospheric turbulence on the mean wind speed along the propagation path.

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